

Black holes and a scalar field in an expanding universe

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Abstract

We consider a model of inhomogeneous universe with the presence of a massless scalar field, where the inhomogeneity is assumed to consist of many black holes. This model can be constructed by following Lindquist and Wheeler, which has already been investigated without the presence of scalar field to show that an averaged scale factor coincides with that of the Friedmann model in Einstein gravity. In this paper we construct the inhomogeneous universe with a massless scalar field, where it is assumed that the averaged scale factor and scalar field are given by those of the Friedmann model including the scalar field. All of our calculations are carried out within the framework of Brans-Dicke gravity. In constructing the model of an inhomogeneous universe, we define the mass of a black hole in the Brans-Dicke expanding universe which is equivalent to the ADM mass in the epoch of the adiabatic time evolution of the mass, and obtain an equation relating our mass with the averaged scalar field and scale factor. As the results we find that the mass has an adiabatic time dependence in a sufficiently late stage of the expansion of the universe, that is our mass is equivalent to ADM mass. The other result is that its time dependence is qualitatively different according to the sign of the curvature of the universe: the mass increases in a decelerating fashion in the closed universe case, is constant in the flat case, and decreases in a decelerating fashion in the open case. It is also noted that the mass in the Einstein frame depends on time. Our results that the mass has a time dependence should be retained even in the general scalar-tensor gravities with a scalar field potential. Furthermore, we discuss the relation of our model of the inhomogeneous universe with the uniqueness theorem of black hole spacetime and the gravitational memory effect of black holes in scalar-tensor gravities.

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1 Introduction

Recently strongly motivated by the supernovae observations [1], general agreement has been reached that our universe is going to turn to accelerated expansion in the context of Friedmann universe model. As one possible model, a scalar field can be introduced in the Friedmann universe [2]. Such a scalar field plays the role as the quintessence of accelerated expansion. The investigation of scalar field for an expanding universe is an important current issue.

On the other hand, our universe consists of celestial objects like galaxies, stars and possibly black holes. In order to draw a more precise picture of our universe than that of the Friedmann model, it is useful to investigate the effects of the inhomogeneities on the expansion of the universe¹. In ref.[4], the expansion law of inhomogeneous universe including many black holes has already been investigated. We call the model of an inhomogeneous universe used in ref.[4] the *cell lattice universe*. It expresses an averaged space time of inhomogeneous universe consisting of many identical black holes, which is constructed as follows: considering a regularly tessellated homogeneous and isotropic universe, each cell of the tessellation is replaced by a spherically symmetric black hole. Here it is assumed that the averaged values of any quantities are defined by the values on the junction surface where a homogeneous universe is connected with a spherical black hole. The behavior of the averaged scale factor is determined by the junction condition of the cell lattice universe, and reproduces the same expansion law that the dust-dominated Friedmann model obeys, as has already been shown in ref.[4]. We believe that the cell lattice universe model gives us the well-defined averaged quantities of an inhomogeneous universe, and that this model coincides with the matter-dominated universe.

In this paper, we extend the cell lattice universe model in order to include a scalar field. Motivated by ref.[4], we assume the averaged scalar field and scale factor are given by the values on the junction surface. Furthermore, we require the averaged quantities to agree with those of the dust-dominated Friedmann model including a scalar field. Here the undetermined quantity is the form of the metric of a spherical black hole. With these assumptions we are interested in the question: *how should the black hole be affected by the expansion of the universe and the existence of a scalar field in order to retain the consistency of the cell lattice universe with the dust-dominated Friedmann model*. That is, in contrast to ref.[4] where the expansion law of the cell lattice universe has been investigated, we turn our interest to the black holes in the cell lattice universe. It is well known that the system of Einstein gravity with scalar fields can be transformed to the framework of scalar-tensor gravity. For simplicity, we consider the scalar field to be a massless one and carry out all of the calculations within the framework of the scalar-tensor gravity, especially of the Brans-Dicke gravity. In constructing the cell lattice universe in Brans-Dicke gravity in the way mentioned at the beginning of this paragraph, we define the mass of the black hole by making use of the undetermined form of the metric. If the time dependence of our mass is adiabatic and the universe is expanding, the mass is equivalent to the ADM mass of a Schwarzschild black hole without regard to the details of the expansion law of the universe. The junction condition in replacing a cell by a black hole gives us an equation relating the mass to the averaged scalar field and

¹ This point of view may lead us to study black holes in a non-flat background. So far only a few works related to this issue have been done numerically or in a toy model [3]

scale factor. Through this equation we can investigate whether and how the mass evolves with time, which is an effect of the expansion of the universe and the existence of the scalar field on black holes.

In sections 2 and 3, Friedmann universe in Brans-Dicke gravity and the method of constructing the cell lattice universe are reviewed, respectively. Section 4 is devoted to the construction of the cell lattice universe in Brans-Dicke gravity and to analyses of the mass which is defined in the section 4. Finally we give a summary and discussion in section 5. Throughout this paper, we set $c = 1$.

2 Friedmann Universe in Brans-Dicke Gravity

The action of Brans-Dicke gravity is

$$S = \int d^4x \sqrt{-g} \left[\varphi R - \frac{\omega}{\varphi} (\nabla \varphi)^2 + \mathcal{L}_m \right], \quad (1)$$

where ω is the constant parameter of this theory, φ is the scalar field coupling to gravity and \mathcal{L}_m is the matter Lagrangian which does not include φ . The effective Newton “constant”, G_{eff} , is related to the scalar field as $\varphi = (16\pi G_{eff})^{-1}$. The field equations derived from this action are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \frac{\omega}{\varphi^2} \left[(\nabla_\mu \varphi)(\nabla_\nu \varphi) - \frac{1}{2}g_{\mu\nu}(\nabla \varphi)^2 \right] - \frac{1}{\varphi} [g_{\mu\nu}\square - \nabla_\mu \nabla_\nu] \varphi + \frac{1}{2\varphi} T_{\mu\nu}, \\ \square \varphi &= \frac{1}{4\omega + 6} T_\mu^\mu, \end{aligned} \quad (2)$$

where $T_{\mu\nu}$ is the energy-momentum tensor derived from \mathcal{L}_m , which is automatically divergenceless: $\nabla_\mu T^{\mu\nu} = 0$.

We consider the Friedmann universe in Brans-Dicke gravity. The metric is spatially homogeneous and isotropic,

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (3)$$

where $k = -1, 0, 1$ and $a(t)$ is the scale factor. The matter in the universe is of perfect fluid type,

$$T_{\mu\nu} = \epsilon u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \quad (4)$$

where u_μ is the 4-velocity of comoving observer, ϵ is the energy density and p is the pressure. Then the field eqs.(2) give three independent equations,

$$H^2 + \frac{k}{a^2} = \frac{\epsilon}{6\varphi} - H \frac{\dot{\varphi}}{\varphi} + \frac{\omega}{6} \left(\frac{\dot{\varphi}}{\varphi} \right)^2, \quad (5)$$

$$\ddot{\varphi} + 3H\dot{\varphi} = \frac{1}{4\omega + 6} (\epsilon - 3p), \quad (6)$$

$$\dot{\epsilon} = -3H(\epsilon + p). \quad (7)$$

where $H = \dot{a}/a$ is the Hubble parameter. This system can be solved provided an equation of state is specified.

For a dust-dominated universe the equation of state is $p = 0$, with which eq.(7) gives $\epsilon = \epsilon_0/a^3$, where ϵ_0 is an integration constant. Then we obtain from eq.(6)

$$\varphi = \frac{\epsilon_0}{4\omega + 6} \int^t dt \frac{t + t_0}{a^3}, \quad (8)$$

where t_0 is an integration constant. Eqs.(5) and (8) determine the time evolution of the scale factor and the scalar field of the dust-dominated Friedmann universe in Brans-Dicke gravity.

For a radiation-dominated universe the equation of state is $p = \epsilon/3$, then eq.(7) gives $\epsilon = \epsilon_0/a^4$, where ϵ_0 is an integration constant. Using eq.(6) we obtain

$$\varphi = \int^t dt \frac{q}{a^3}, \quad (9)$$

where q is an integration constant. Eqs.(5) and (9) determine the time evolution of the scale factor and the scalar field of the radiation-dominated Friedmann universe in Brans-Dicke gravity.

For the use of section 4, let us review the frame transformation. By a suitable conformal transformation of the metric $g_{\mu\nu}$ to the other one $\tilde{g}_{\mu\nu}$, the action S of eq.(1) can be treated as Einstein gravity with a massless scalar field which does not couple to gravity. Such a transformation is given by

$$g_{\mu\nu} = \exp \left[-\frac{\sigma}{2\omega + 3} \right] \tilde{g}_{\mu\nu}, \quad (10)$$

where σ is a scalar field defined by $\varphi = (16\pi G_0)^{-1} \exp[\sigma/(2\omega + 3)]$. Here G_0 is the ordinary Newton constant in the Einstein gravity. Then the action S becomes

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{16\pi G_0} (\tilde{R} - (\tilde{\nabla}\sigma)^2) + e^{-2\sigma/(2\omega+3)} \mathcal{L}_m \right]. \quad (11)$$

The action expressed by $\tilde{g}_{\mu\nu}$ is said to be in the Einstein frame, while the action of eq.(1) is in the original frame. We add tilde to quantities in the Einstein frame. It is obvious by this frame transformation that the Brans-Dicke gravity in the original frame becomes the Einstein gravity in the limit $\omega \rightarrow \infty$, that is, $g_{\mu\nu} = \tilde{g}_{\mu\nu}$.

It has already known analytically that, in the dust-dominated and flat universe case, $p = 0$ and $k = 0$, only the choice that $t_0 = 0$ lets the scale factor $a(t)$ in Brans-Dicke gravity go over smoothly to that of Einstein gravity in the limit $\omega \rightarrow \infty$ [5]. With $t_0 = 0$ we can find the relation

$$\frac{8\omega + 12}{3\omega + 4} = \frac{\epsilon_0 t^2}{\varphi a^3}. \quad (12)$$

The scale factor and scalar field in this case can be easily obtained to be

$$\begin{aligned} a(t) &= a_0 t^{(2\omega+2)/(3\omega+4)}, \\ \varphi(t) &= \varphi_0 t^{2/(3\omega+4)}, \end{aligned} \quad (13)$$

where a_0 and φ_0 are constants. In the other curvature cases $k = \pm 1$ with a restriction $t_0 = 0$, eq.(5) indicates that the universe is dominated at early times by terms other than the curvature one k/a^2 . That is, the curved universe is effectively flat at early times.

3 Cell Lattice Universe

3.1 Strategy to construct the cell lattice universe and Averaged quantities

It is a purely geometric problem to construct the cell lattice universe which expresses an averaged inhomogeneous expanding universe including many identical spherically symmetric objects [4]. The procedure of the construction consists of two steps as follows: *1st step*, regularly tessellate the spherical, flat or hyperbolic spatial section, Σ_t , of a homogeneous and isotropic universe by a regular polyhedron [6] [7]. *2nd step*, replace each cell made of a polyhedron by a metric of the spherically symmetric object. For the junction condition in the 2nd step we consider the radially directional differential of any great circle's circumference on the two dimensional spherical junction surface in Σ_t , which can be expressed as:

$$D = \frac{d(\text{Circumference of the junction surface in } \Sigma_t)}{d(\text{Physical radial distance along } \Sigma_t)}. \quad (14)$$

The junction condition is to connect such a differential measured in the homogeneous and isotropic universe, D_{univ} , with that measured in the metric of a spherically symmetric object, D_{BH} . That is the junction condition is given by

$$D_{univ} = D_{BH}. \quad (15)$$

Furthermore, in the 2nd step, there are some regions where spherically symmetric metrics overlap, and other regions where the metrics do not cover. Here we assume that the averaged values of any quantities are defined by the values on the spherical junction surface. For example, the averaged scale factor of the cell lattice universe is the scale factor on the spherical junction surface. The averaged cosmological time is set by the proper time of the observer staying on the spherical junction surface. We consider the cell lattice universe to be an averaged spacetime in such a sense.

3.2 Construction of the cell lattice universe

The metric of a homogeneous and isotropic universe can be given in the same form as eq.(3). The 1st step in constructing the cell lattice universe is to determine the radius of each spherical cell measured in this metric [6] [7] by requiring that the volume of a spherical cell coincides with that of a regular polyhedron. There are some regular polyhedra which are candidates for the regular tessellation for each curvature case. The types of polyhedra are listed later in this section after the explanation of the junction condition. We denote the comoving radius of the spherical cell by r_c . Since the comoving number density of cells is constant, the cell expands comovingly. Therefore $r_c = \text{const.}$, and the averaged cosmological time is identical to the time coordinate of the metric (3), t_c , on the junction surface $r = r_c$. Hereafter we denote the physical radius of a cell by $l_c(t_c) = a(t_c)r_c$.

In proceeding to the 2nd step, we define precisely the “radial direction” in the differential of the great circle on the junction surface. In choosing the parameter along the radial direction, χ , into a coordinate

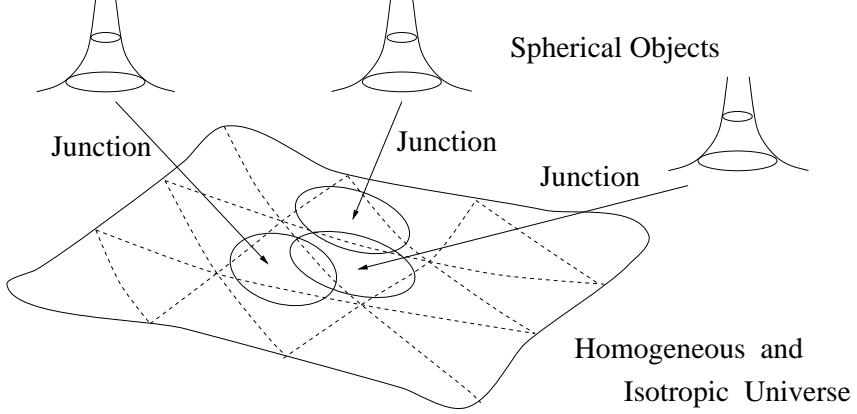


Figure 1: Graphical image of the cell lattice universe. The region surrounded by broken lines is the regular polyhedron. The circle of real lines is the spherical cell. Each cell on the homogeneous and isotropic universe is replaced by a spherically symmetric object. There are some region where some objects overlap, while the other regions are uncovered.

system such as (t, χ, θ, ϕ) , we define the coordinate parameter χ as follows: with the conditions that t, θ and ϕ are constant, the line element along the radial direction is given by $ds^2 = a^2 d\chi^2$, that is to say, χ is the ‘‘comoving radial distance’’. For convenience we transform the coordinate (t, r, θ, ϕ) to (t, χ, θ, ϕ) by $r = f_k(\chi)$, where $f_k(\chi) = \sin \chi$, χ and $\sinh \chi$ for $k = 1, 0$ and -1 , respectively:

$$ds^2 = -dt^2 + a(t)^2 \left[d\chi^2 + f_k(\chi) (d\theta^2 + \sin \theta d\phi^2) \right]. \quad (16)$$

In denoting $r_c = f_k(\chi_c)$, this χ_c should be constant because χ_c is the comoving coordinate on the junction surface. The circumference of great circle, C_{gc} , is calculated in this metric to be

$$C_{gc} = \int_{\Sigma_t, \chi=\chi_c, \theta=\pi/2} \sqrt{g_{\phi\phi}} d\phi = 2\pi a f_k(\chi_c) \quad (= 2\pi l_c). \quad (17)$$

Then we obtain the radially directional differential of the great circle in the metric (16), D_{univ} , to be

$$D_{univ} = \frac{dC_{gc}}{d(a\chi)} \Big|_{\Sigma_t, \chi_c} = 2\pi \frac{df_k(\chi_c)}{d\chi_c}. \quad (18)$$

The metric of a spherically symmetric object can be given by

$$ds^2 = -A(T, R) dT^2 + B(T, R) dR^2 + R^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (19)$$

where $A(T, R)$ and $B(T, R)$ are the arbitrary independent functions of T and R . The radial coordinate R is the physical radial length. Hereafter we assume that the radius of a spherical object is smaller than that of a cell, therefore the metric (19) is the exterior spacetime of the object. In the 2nd step of

constructing the cell lattice universe, we should pay attention to the fact that the coordinates (T, R) of spherical object are not necessarily identical to (t, r) or (t, χ) of a homogeneous and isotropic universe. When the origins of the coordinates of both metrics coincide, the radii of the junction surface in both coordinate systems coincide. That is, we have $R(t_c) = l_c(t_c)$ on the junction surface. The time T of the metric (19) on the junction surface can be expressed by a function of t_c and r_c as $T_c = T(t_c, r_c)$, which should reflect the junction condition and the time evolution of the universe. The radially directional differential of the great circle in the metric (19) should be calculated on Σ_t . The circumference of the great circle, C_{gc} , is given by eq.(17). At the junction surface, the vector which is orthogonal to Σ_t can be given by $e^\mu = (dT_c, dl_c, 0, 0)$ in the coordinate of metric (19), then the vector which is orthogonal to the junction surface and parallel to Σ_t can be obtained to be $v^\mu = n(-g^{TT}dl_c, g^{RR}dT_c, 0, 0)$, where n is the normalization constant. This gives us the difference along the radial direction as $d(a\chi)^2 = g_{\mu\nu}v^\mu v^\nu$. Here we determine the normalization, n , by requiring that the difference of the circumference be related to the vector v^μ as $dC_{gc} = 2\pi dl_c = 2\pi v^R$. For the averaged cosmological time, we have the relation $-dt_c^2 = g_{\mu\nu}e^\mu e^\nu$. Then we can calculate the radially directional differential of the great circle in the metric (19), which we denote as D_{BH} , as follows:

$$D_{BH} = \frac{2\pi dl_c}{d(a\chi_c)} = 2\pi \frac{g^{RR}dT_c}{\sqrt{g_{\mu\nu}v^\mu v^\nu}} = 2\pi \sqrt{-\frac{g_{TT}}{g_{RR}}} \frac{dT_c}{dt_c} \quad (20)$$

With the preparations done above, we proceed to calculating the junction condition, $D_{univ} = D_{BH}$. Substituting eq.(18) and the second expression of D_{BH} of eq.(20) for the junction condition, we find the relation: $(dT_c/dl_c)^2 = \alpha_k^2 B/[A(\alpha_k^2 - 1/B)]$, where $\alpha_k = \cos \chi_c$, 1 and $\cosh \chi_c$, for $k = +1$, 0 and -1 , respectively. With this relation and $D_{univ} = D_{BH}$ with the third expression of D_{BH} in eq.(20), we obtain the equation [4]:

$$\left(\beta_k \frac{da(t_c)}{dt_c} \right)^2 = \alpha_k^2 - \frac{1}{B(t_c, l_c)}, \quad (21)$$

where $\beta_k = f_k(\chi_c)$. The parameters α_k and β_k are determined by the cell radius χ_c . Here we list the types of regular polyhedra, χ_c and N in the Table 1, where N is the number of the cells included in the spatial surface Σ_t [6] [7].

For the cell lattice universe made using Schwarzschild black holes in Einstein gravity, the averaged scale factor is given by eq.(21) with $B^{-1} = 1 - 2G_0M/R$, where G_0 is the ordinary Newton constant.

It can be easily checked that, without regard to the polyhedron used for the tessellation, this equation reproduces the same expansion law as for a dust-dominated Friedmann universe in Einstein gravity [4]. In this paper we assume that, even in any of the generalized theories of gravity, the averaged values of any quantities coincide with those in the dust-dominated Friedmann model.

4 Time evolution of Black Holes in an Expanding Universe

In this section we consider the spherical object in the cell lattice universe to be a black hole.

Curvature k	Polyhedron	χ_c	N
+1	Tetrahedron	1.057	5
+1	Tetrahedron	0.6866	16
+1	Tetrahedron	0.1993	600
+1	Cube	0.8832	8
+1	Octahedron	0.5951	24
+1	Dodecahedron	0.3426	120
0	Cube	arbitrary	∞
-1	Cube	0.7185	∞
-1	Dodecahedron	1.251	∞
-1	Dodecahedron	0.9505	∞
-1	Icosahedron	0.9747	∞

Table 1: List of the polyhedra for the tessellation and the radii χ_c corresponding to each polyhedron. The fourth column N is the number of spherical objects included in the spatial surface Σ_t . [6] [7]

4.1 Black hole mass and cell lattice universe

As mentioned in section 1, our interest in this paper is the effect of the expansion of the universe and a massless scalar field on black holes. We consider this issue within the framework of Brans-Dicke gravity and try to extract such effects from the cell lattice universe which is assumed to be meaningful as an approximation of multi-black hole spacetime even in Brans-Dicke gravity. This issue can be discussed by eq.(21) once the function $B(T, R)$ is specified.

To specify the form of $B(T, R)$, we should note here that the uniqueness theorem [8] of the black holes in Brans-Dicke gravity has already been established with the asymptotically flat condition, not in the expanding universe [9]. That is, the spherically symmetric non-rotating black hole in the asymptotically flat spacetime is of the Schwarzschild black hole even in Brans-Dicke gravity. Furthermore, it is important for specifying $B(T, R)$ to note the issue peculiar to the scalar-tensor gravity: *gravitational memory effect*. This issue was originally recognized in ref.[10] and expressed as follows: when a black hole is formed in the expanding universe, does the scalar field on the event horizon keep the value at the black hole formation time during cosmological evolution? If the scalar field keeps its original value, it is said that the black hole has a gravitational memory effect, if not, the black hole does not have the memory ². Concerning this issue, ref.[11] treats the perturbation of a stationary black hole with the boundary condition that the scalar field is proportional to the time in the region spatially far from the black hole. With this model, it is suggested in ref.[11] that the gravitational memory effect does not occur in scalar-tensor gravities. That is, the scalar field on the event horizon seems to have a time dependence along with cosmological evolution. Then we adopt a naively acceptable ansatz as follows:

² In ref.[10], two possible scenarios of cosmology are discussed: the scenario that primordial black holes have a gravitational memory effect, and the other scenario where primordial black holes don't have such effects.

the spherically symmetric black hole composing the cell lattice universe in Brans-Dicke gravity is of the same form as the Schwarzschild black hole except for the one point that its radius depends on the cosmological time, $R_g(t_c)$. With this ansatz we define the mass of the black hole in the cell lattice universe, M , in Brans-Dicke gravity ³ as,

$$M(t_c) \equiv 8\pi\varphi(t_c)R_g(t_c). \quad (22)$$

That is, we assume the black hole to be of “Schwarzschild-type” given by eq.(19) on the junction surface with

$$A(t_c, l_c) = B(t_c, l_c)^{-1} = 1 - \frac{M(t_c)}{8\pi\varphi(t_c)l_c(t_c)}. \quad (23)$$

This mass, M , certainly has the dimensions of mass, but it is not obvious whether this definition can be consistent with the ADM mass defined by the generator of Killing time translation in a stationary space time. Without regard to the details of the expansion law of the averaged scale factor, provided that M changes adiabatically over a cosmological time scale and that the black hole horizon size is much smaller than the cosmological one, we can consider that the black hole exists in a local asymptotically flat region as for an ordinary Schwarzschild black hole and that the cosmological time t_c is equivalent to the time T of the metric (19) within the scale of the black hole. In such a case our mass M can be effectively treated as the ADM mass defined by the generator of the local asymptotically timelike Killing vector, where the Killing time is T of the metric (19).

With specifying $B(T, R)$ as above, the junction condition (21) gives

$$M(t_c) = 8\pi\beta_k^3\varphi(t_c)a(t_c)\left[\dot{a}(t_c)^2 + k\right]. \quad (24)$$

Clearly, since β_k is only related to the normalization of the mass, the behavior of $M(t_c)$ is not affected by the value of χ_c , that is, it is not affected by the choice of the polyhedron for the tessellation in the 1st step of the construction of the cell lattice universe. Here, motivated by the comment in the final paragraph in the previous section, we assume the cell lattice universe to be a well-defined averaged model of an inhomogeneous universe, reproducing the expansion law of a dust-dominated Friedmann model even in Brans-Dicke gravity. Because of this assumption, the averaged scale factor, $a(t_c)$, and the scalar field, $\varphi(t_c)$, are given by the dust-dominated Friedmann universe in Brans-Dicke gravity. The time evolution of $M(t_c)$ can be investigated using eqs.(5), (8) and (24). It is not *a priori* obvious whether or not the mass M depends adiabatically on the cosmological time.

As has already been pointed out in ref.[11], provided our mass M can be considered as the ADM mass in a local asymptotically flat region, we can discuss whether the mass in the Einstein frame changes with time. Attaching a tilde to the quantity in the Einstein frame, we find the relation $M/\tilde{M} = d\tilde{T}/dT$, because the ADM mass is given by the generator of the local asymptotic Killing vector, which is normalized at the boundary of the local asymptotically flat region and scales inversely

³ There have already been some discussions concerning the spatial profile of the scalar field and the mass of a black hole and boson star in the scalar-tensor gravity, see [12] for examples.

with the time T . Since the Killing time translation is given by the line element along integral curves of the timelike Killing vector, we obtain the relation at the boundary of the local asymptotically flat region: $-dT^2 = -\exp[-\sigma/(2\omega + 3)] d\tilde{T}^2$, where dT and $d\tilde{T}$ express the Killing time translations in the Brans-Dicke and the Einstein frames, respectively, and the frame transformation given by eq.(10) is used. Therefore we obtain the relation of the masses [11]

$$M = \exp\left[\frac{\sigma}{4\omega + 6}\right] \tilde{M} \propto \sqrt{\varphi} \tilde{M}. \quad (25)$$

If \tilde{M} is constant, M is proportional to $\sqrt{\varphi}$. In the case that the M defined by eq.(24) can be treated as the ADM mass, by comparing the M with $\sqrt{\varphi}$, we can obtain a suggestion concerning the time dependence of \tilde{M} which is the black hole mass in an expanding universe in Einstein gravity with the presence of a scalar field.

4.2 Analysis of the black hole mass

In the numerical calculations shown here, our interests are the time dependence of M , not the absolute value of it. That is, we investigate whether or not M changes adiabatically on a cosmological time scale and whether M is proportional to $\sqrt{\varphi}$ or not, since these issues reveal that our black hole mass in the Brans-Dicke frame is equivalent to the ADM mass and that the black hole mass in the Einstein frame has a time dependence. Hereafter we denote the averaged cosmological time t_c simply by t .

In flat case $k = 0$, it is easily found from eqs.(13) and (24) that the mass M is constant, and that the radius $R_g(t) \propto 1/\varphi(t)$ decreases in a decelerating fashion, while the averaged scale factor continues to expand. Therefore, the mass can be effectively treated as ADM mass after the universe expands enough to be much larger than the black hole. Clearly $M \not\propto \sqrt{\varphi}$. This means that the mass in the Einstein frame has a time dependence.

In two cases $k = \pm 1$, we calculate the mass $M(t)$ numerically using *Mathematica* from $t = 0.5$ to $t = 50000$ for the open case $k = -1$, and to $t = 1000$ for the closed case $k = 1$. We attach the subscript “ i ” to the quantities evaluated at $t = 0.5$, and the subscript “ f ” at $t = 50000$ for $k = -1$ and at $t = 1000$ for $k = 1$. These calculation time intervals of calculation should correspond to the dust-dominated era of our universe, since the cell lattice universe is assumed to be the averaged spacetime of a dust-dominated inhomogeneous universe as discussed in the previous subsection. We regard the end time of the calculation as our present time, because, as shown in the following, the rate of change of the mass M in time evolution is sufficiently small in comparison with the averaged Hubble parameter at the end time for both cases $k = \pm 1$, that is M evolves adiabatically. Here the averaged Hubble parameter $H = \dot{a}/a$ is defined by using the averaged scale factor. As mentioned at the end of section 2, the curved Friedmann universe can be effectively treated as being flat in the early stage of the time evolution. Therefore, we set the integration constant t_0 of eq.(8) to zero ($t_0 = 0$) in order to reproduce the expansion law of the Friedmann model in Einstein gravity as $\omega \rightarrow \infty$. The initial values of the averaged scale factor a_i and scalar field φ_i are related through eq.(12). Our choices of the other parameters are as follows: the initial value of scale factor, $a_i = 10$. The Brans-Dicke parameter

Time t	0.5	2	10	20	100	500	2000	5000	10000	30000
Red Shift z	5559	1840	601.8	375.8	125.5	40.42	14.06	6.376	3.170	0.5964

Table 2: Table of the averaged cosmological time and red shift for the open case $k = -1$. The present time is $t = 50000$.

$\omega = 500$, which is the experimental lower bound of ω . Another integration constant $\epsilon_0 = 100$. The size of one cell in the cell lattice universe $\chi_c = 0.9747$ (Icosahedron) and 0.1993 (Tetrahedron) for $k = -1$ and 1 , respectively [6] [7]. The value of χ_c does not affect the behavior of M as shown in eq.(24).

For the open case $k = -1$, the averaged Hubble time at present $t = 50000$ is $1/H_f = 53931$. The table 2 shows some values of the averaged cosmological red shift which is defined by $z(t) = a_f/a(t) - 1$ with the averaged scale factor a . Fig.2 is for the rate of change of $M(t)$ normalized by the averaged Hubble parameter: $H_m(t) = (\dot{M}/M)/H$. The H_m is of negative and asymptotes to zero for sufficiently late stage of cosmological evolution $t > 10000$ where $z < 3.170$. The rate of change, for example at $t = 2$, takes the value $H_m(2) = 0.0001469$. This indicates that the time evolution of M is less significant than that of the averaged scale factor at least for $t > 2$ ($z < 1840$), where $H_m < O(10^{-4})$. That is, M is adiabatic in this epoch. The radius of black hole R_g decreases in a decelerating fashion with time, from $R_{gi} = 5210$ to $R_{gf} = 5142$, while the averaged scale factor continues to expand for all time. Therefore, the mass M can be effectively treated as being ADM mass after the universe expands enough to be much larger than the black hole. Fig.3 is the plot of M with SqM which is defined by $SqM(t) = M_f \sqrt{\varphi(t)/\varphi_f}$. The square root of the averaged scalar field SqM at the initial time is $SqM_i = 609.4$, and increases in a decelerating fashion for all time. The rate of change of mass H_m becomes zero at $t = 14.92$. The mass M increases from $M_i = 613.6$ to $M(14.29) = 613.9$, and decreases in a decelerating fashion after the time $t = 14.92$ where $z(14.92) = 17.25$. Fig.3 indicates that mass M does not coincide with SqM , and that the mass in the Einstein frame has a time dependence.

For the closed case $k = 1$, the averaged Hubble time at present $t = 1000$ is $1/H_f = 1743$. Table 3 includes some values of the averaged cosmological red shift $z(t) = a_f/a(t) - 1$. Fig.4 shows H_m , the rate of change of M normalized by the averaged Hubble parameter. The rate of change H_m decreases rapidly at early times, and turns to increase around $t = 20$ where $z = 11.09$. At present, the rate of change is $H_{mf} = 0.0006698$. The increase of H_m continues up to the turning time of the averaged scale factor from expansion to contraction, where the turning time is $t = 5483$. The rate of change, for example at $t = 2$, is $H_m(2) = 0.0001565$. This indicates that the time evolution of the mass M is adiabatic within the epoch from $t = 2$ to the present $t = 1000$, where $H_m < O(10^{-4})$. The black hole radius R_g decreases in a decelerating fashion with time, from $R_{gi} = 27.54$ to $R_{gf} = 27.29$, while the averaged scale factor lasts to expand until the turning time $t = 5483$. Therefore, the mass M can be effectively treated as ADM mass after the universe expands to a size much larger than the black hole. Fig.5 shows a plot of M and SqM . The square root of the averaged scalar field SqM increases in a decelerating fashion for all time from the initial value $SqM_i = 3.227$. The mass M takes the initial value $M_i = 3.224$, and also

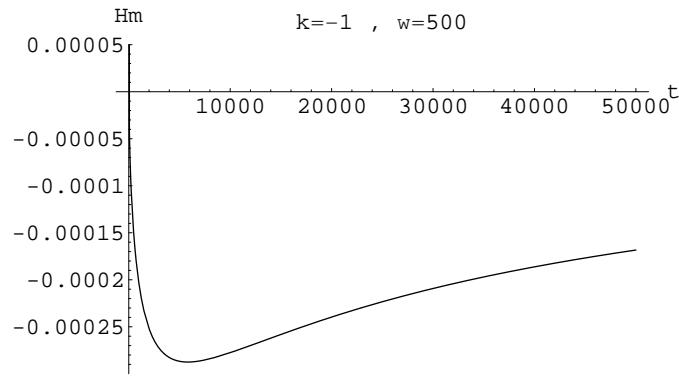


Figure 2: Graph of $H_m = (\dot{M}/M)/H$ for $k = -1$, the change rate of the mass. $H_{mi} = 1.00283$, $H_m(2) = 0.0001469$ and $H_m(14.92) = 0$.

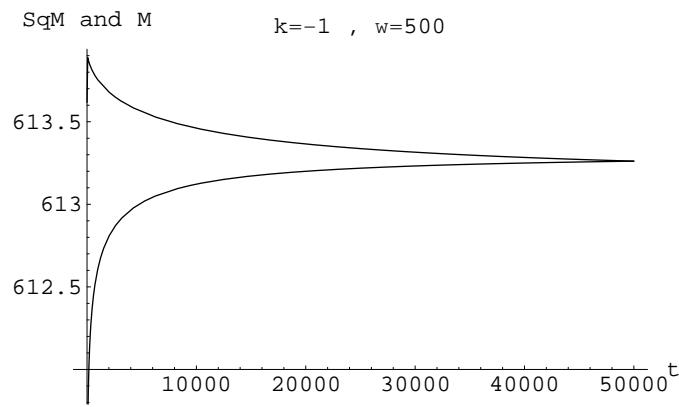


Figure 3: Graph of $M(t)$ and $SqM(t)$ for $k = -1$. The upper curve is for M while the bottom one for SqM . In this graph the overall scale of SqM is set so that $M_f = SqM_f$. $M_i = 613.6$, $M(14.29) = 613.9$ and $SqM_i = 609.4$.

Time t	0.5	2	5	10	15	20	50	100	300	500
Red Shift z	174.4	57.27	29.73	18.21	13.64	11.09	5.592	3.189	1.072	0.5085

Table 3: Table of the averaged cosmological time and red shift for the closed case $k = 1$. The present time is $t = 1000$.

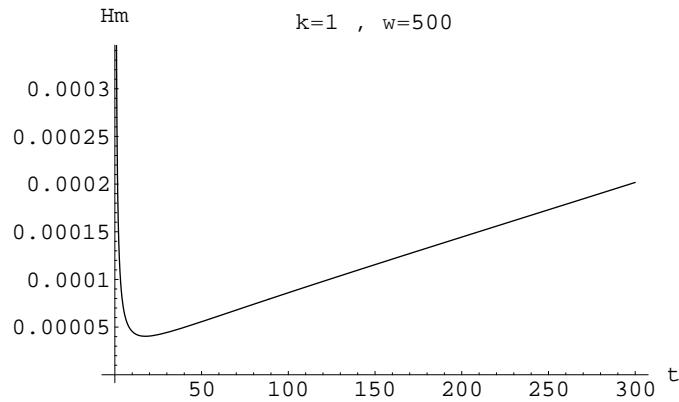


Figure 4: Graph of $H_m = (\dot{M}/M)/H$ for $k = 1$, the change rate of the mass. $H_{mi} = 1.00282$, $H_m(2) = 0.0001565$ and $H_{mf} = 0.0006698$.

increases in a decelerating fashion for all time. Fig.5 indicates that M and SqM do not coincide with each other, and that the mass in the Einstein frame has a time dependence.

5 Summary and Discussion

In order to investigate the effects of expansion of the universe and a scalar field on the celestial objects which compose the inhomogeneity of the universe, we constructed the cell lattice universe in Brans-Dicke gravity. The assumptions in constructing the universe were that the black holes included in the universe are of “Schwarzschild-type”, eqs.(19) and (23), and that the averaged scale factor and scalar field are given by those of the Friedmann universe. Furthermore, we defined the mass M by eq.(22) which is equivalent to the ordinary ADM mass in the case of adiabatic time evolution of M . The junction condition of the cell lattice universe gives eq.(24) to calculate the time evolution of the black hole mass M . It is not *a priori* known whether and how the mass depends on time.

As a result, it turns out that M behaves in a qualitatively different way with respect to the value of k . The mass M decreases in a decelerating fashion for the open case $k = -1$, stays completely constant for the flat case $k = 0$ and increases in a decelerating fashion for the closed case $k = 1$. The rate of change of the mass H_m is very small for both cases of $k = \pm 1$, $H_m < O(10^{-4})$ with our numerical

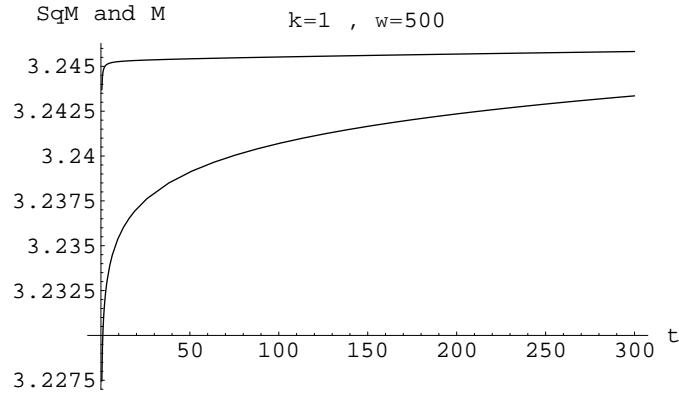


Figure 5: Graph of $M(t)$ and $SqM(t)$ for $k = 1$. The upper curve is for M while the bottom one for SqM . In this graph the overall scale of SqM is set so that $M_f = SqM_f$. $M_i = 3.244$ and $SqM_i = 3.227$.

results. This means that M evolves adiabatically in time. Because here we assume that the averaged scale factor in eq.(24) is given by that of the Friedmann model in Brans-Dicke gravity, the adiabaticity of M is consistent with the fact that the cell lattice universe constructed with the Schwarzschild black hole, whose mass is completely constant, reproduces the expansion law of the Friedmann universe in Einstein gravity [4]. Furthermore, the radius R_g decreases in a decelerating fashion for every case of $k = \pm 1, 0$, while the averaged scale factor continues to increase. This behavior of R_g can be easily understood for the case $k = 0$, which is $R_g \propto t^{-2/(3\omega+4)}$. Therefore, it is reasonable to consider the black hole is in a local asymptotically flat region after the universe expands enough to be much larger than the black hole, consequently mass M is equivalent to the ADM mass defined by the local asymptotic Killing time translation.

Furthermore, we can recognize using eq.(25) that the black hole mass in the Einstein frame has a time dependence for every case of $k = \pm 1, 0$. According to the uniqueness theorem [8] [9], non-rotating and non-charged black hole in asymptotically flat spacetime is specified only by the ADM mass, which is completely constant in this theorem. The time dependence of the mass may indicate that the uniqueness theorem in an expanding universe will be broken on a cosmological time scale. It is a very interesting problem which remains to be solved.

In the numerical calculations shown in the section 4, we set $\omega = 500$ which is the lowest experimental value. If the parameter is set as $\omega < 500$, the absolute value of the rate of change of the mass M tends to increase. However the total behavior of the mass M and the radius R_g for the case $\omega < 500$ are the same as for the case $\omega = 500$. Consequently, for arbitrary value of ω , the mass can be equivalent to the ADM mass once the universe expands enough to be much larger than the black hole size.

So far we have only treated the Brans-Dicke gravity as a representative case of scalar-tensor gravities in which the parameter ω depends on the scalar field, and have not introduced any potential of the scalar field. When, instead of Brans-Dicke gravity, we consider a general scalar-tensor gravity with a

potential of the scalar field in constructing the cell lattice universe, the averaged scale factor and scalar field may behave quite differently from those in Brans-Dicke gravity. However, the same arguments and results obtained until the previous paragraph are true of this case if the averaged quantities asymptote to those in the Brans-Dicke gravity at least in a sufficiently late stage of the expansion of the universe. Furthermore, in the case that the averaged scale factor or scalar field in a general theory of gravity always behaves differently from that in the Brans-Dicke gravity, though we need to reanalyze eq.(24) in order to know the details of the time dependence of M , however, it is natural to propose that the mass M depends on time in this case too. That is, it seems to be quite a general point that the black hole mass in expanding universe with the presence of scalar field has some time dependence.

Let us comment on our assumption, i.e. the “Schwarzschild-type” ansatz of the black hole composing the cell lattice universe given by eqs.(19), (22) and (23). The construction of the cell lattice universe where a cell is replaced by a black hole, means that the dust-matter on homogeneous and isotropic universe in a cell is concentrated at a center point of the cell. Therefore, it is appropriate to consider the black hole spacetime replacing the cell includes only the scalar field coupling with gravity in the Brans-Dicke frame. The field equation of such a scalar field is obtained by substituting $T_{\mu\nu} = 0$ into eqs.(2),

$$G_{\mu\nu} = \frac{\omega}{\varphi^2} \left[(\nabla_\mu \varphi)(\nabla_\nu \varphi) - \frac{1}{2} g_{\mu\nu} (\nabla \varphi)^2 \right] + \frac{1}{\varphi} \nabla_\mu \nabla_\nu \varphi, \quad (26)$$

where the non-zero components of the Einstein tensor are calculated from eqs.(19), (22) and (23),

$$\begin{aligned} G_{01} &= \frac{\dot{R}_g}{R^2 - R \cdot R_g}, \\ G_{22} &= -R^3 \left[\frac{\ddot{R}_g}{2(R - R_g)^2} + \frac{\dot{R}_g^2}{(R - R_g)^3} \right], \\ G_{33} &= \sin^2 \theta G_{22}, \end{aligned} \quad (27)$$

where $\dot{R}_g = dR_g/dT$ with the time T of metric (19). The scalar field on the Schwarzschild-type spacetime should satisfy the above field eqs.(26) and (27).

Our ansatz of the Schwarzschild-type black hole is motivated by the uniqueness theorem of the black hole in asymptotically flat spacetime [9] and the indication in ref.[11] that the gravitational memory effect seems not to occur. Conversely, in paying attention to the latter motivation, the gravitational memory effect can be discussed using eqs.(26) and (27) with the boundary condition $\varphi(T)$ at $R = l_c(T)$ given by the scalar field of Friedmann model, where $l_c(T)$ is determined by the junction of our cell lattice universe. By investigating this system we can know whether or not the scalar field φ has a spatial dependence. If not, it means that the gravitational memory effect does not occur in Brans-Dicke gravity, and that the indication in ref.[11] and our discussion in this paper are supported. The model of spacetime in such an approach to the gravitational memory effect, can be considered as a modified *swiss cheese universe* [13]. In the ordinary swiss cheese model, a spherically symmetric region in the Friedmann universe is replaced by an ordinary Schwarzschild black hole of constant radius and mass,

but here we replace the spherical region by a “Schwarzschild-type” black hole. This is an interesting and solvable problem.

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